Mathematics
Paper 3 (Calculator)
Higher Tier

You must have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser, calculator. Tracing paper may be used.

Instructions

- Use black ink or ball-point pen.
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all questions.
- Answer the questions in the spaces provided — there may be more space than you need.
- You must show all your working.
- Diagrams are NOT accurately drawn, unless otherwise indicated.
- Calculators may be used.
- If your calculator does not have a π button, take the value of π to be 3.142 unless the question instructs otherwise.

Information

- The total mark for this paper is 80
- The marks for each question are shown in brackets — use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.
Answer ALL questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 (a) Expand and simplify \((x + 5)(x - 9)\)

(b) Factorise fully \(9x^2 + 6x\)

(Total for Question 1 is 4 marks)

2 (a) Use your calculator to work out \(\frac{29^2 - 4.6}{\sqrt{35} - 1.9^3}\)

Write down all the figures on your calculator display.

(b) Write your answer to part (a) correct to 4 significant figures.

(Total for Question 2 is 3 marks)
3  The scatter graph shows information about the marks a group of students got in a Science test and in a Maths test.

Jamie got a mark of 34 in the Science test.

Using the scatter graph, find an estimate for Jamie’s mark in the Maths test.

(Total for Question 3 is 2 marks)
The table gives information about the times taken, in seconds, by 18 students to run a race.

<table>
<thead>
<tr>
<th>Time ($t$ seconds)</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5 &lt; t \leq 10$</td>
<td>1</td>
</tr>
<tr>
<td>$10 &lt; t \leq 15$</td>
<td>2</td>
</tr>
<tr>
<td>$15 &lt; t \leq 20$</td>
<td>7</td>
</tr>
<tr>
<td>$20 &lt; t \leq 25$</td>
<td>8</td>
</tr>
</tbody>
</table>

Work out an estimate for the mean time.
Give your answer correct to 3 significant figures.

............... seconds

(Total for Question 4 is 3 marks)
5 Write 37 cm$^3$ in mm$^3$

...................................................... mm$^3$

(Total for Question 5 is 1 mark)

6 Nimer was driving to a hotel. He looked at his Sat Nav at 13:30

<table>
<thead>
<tr>
<th>Time</th>
<th>13:30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance to destination</td>
<td>65 miles</td>
</tr>
</tbody>
</table>

Nimer arrived at the hotel at 14:48

Work out the average speed of the car from 13:30 to 14:48
You must show all your working.

...................................................... mph

(Total for Question 6 is 4 marks)
7  (a) Write 32460000 in standard form.

(b) Write 4.96 \times 10^{-3} as an ordinary number.

Asma was asked to compare the following two numbers.

\[ A = 6.212 \times 10^8 \quad \text{and} \quad B = 4.73 \times 10^9 \]

She says,

\[ \text{“6.212 is bigger than 4.73 so } A \text{ is bigger than } B.” \]

(c) Is Asma correct?

You must give a reason for your answer.

(Total for Question 7 is 3 marks)
8 The diagram shows a regular pentagon and a parallelogram.

Work out the size of the angle marked \( x \).
You must show all your working.

\[
\begin{align*}
\text{Diagram with angle } 117^\circ \\
\text{and angle } x
\end{align*}
\]

(Total for Question 8 is 4 marks)
Enlarge triangle A by scale factor 2.5 with centre (0, 1)

(Total for Question 9 is 2 marks)
10 (a) Solve \( \frac{9 + x}{7} = 11 - x \)

\[ x = \quad \]  

(b) Simplify \( \frac{4(y + 3)^3}{(y + 3)^2} \)

\[ \quad \]  

(Total for Question 10 is 4 marks)
11 The probability tree diagram shows the probabilities that Bismah will be late for work on two days next week.

Calculate the probability that Bismah will be late on exactly one of the two days.

(Total for Question 11 is 3 marks)
The stem and leaf diagram shows information about the heights, in cm, of 23 sunflowers.

```
17 | 3 4 9
18 | 6 8 8
19 | 0 0 1 4 6 7 8
20 | 1 4 7 7 9 9
21 | 4 8 8 9
```

Key: 17|3 represents 173 cm

On the grid, draw a box plot for this information.

(Total for Question 12 is 3 marks)
13 Liquid A and liquid B are mixed together in the ratio 2:13 by volume to make liquid C.

Liquid A has density 1.21 g/cm$^3$
Liquid B has density 1.02 g/cm$^3$

A cylindrical container is filled completely with liquid C.
The cylinder has radius 3 cm and height 25 cm.

Work out the mass of the liquid in the container.
Give your answer correct to 3 significant figures.
You must show all your working.

\[ \text{Mass of liquid} = \text{Volume} \times \text{Density} \]

\[ \text{Volume} = \pi r^2 h \]

\[ \text{Mass} = \pi \times 3^2 \times 25 \times (1.21 + \frac{13}{15} \times 1.02) \]

\[ = \pi \times 9 \times 25 \times 1.21 \times \frac{28}{15} \]

\[ = 706.8 \text{ g} \]

(Total for Question 13 is 4 marks)
14 A group of people went to a restaurant.
Each person chose one starter and one main course.

<table>
<thead>
<tr>
<th>starter</th>
<th>main course</th>
</tr>
</thead>
<tbody>
<tr>
<td>soup</td>
<td>lasagne</td>
</tr>
<tr>
<td>prawns</td>
<td>curry</td>
</tr>
</tbody>
</table>

the number of people who chose soup : the number of people who chose prawns = 2 : 3

Of those who chose soup,
the number of people who chose lasagne : the number of people who chose curry = 5 : 3

Of those who chose prawns,
the number of people who chose lasagne : the number of people who chose curry = 1 : 5

What fraction of the people chose curry?
You must show how you get your answer.
15 Prove algebraically that the sum of the squares of any two consecutive even numbers is always a multiple of 4

(Total for Question 15 is 3 marks)

16 \( y \) is inversely proportional to the square of \( x \).

\[ y = 8 \text{ when } x = 2.5 \]

Find the negative value of \( x \) when \( y = \frac{8}{9} \)

(Total for Question 16 is 3 marks)
17 Here is the graph of \( y = x^2 - 3 \)

Use the graph to find estimates for the solutions to the equation \( x^2 - 2x - 2 = 0 \)

You must show how you get your solutions.

(Total for Question 17 is 4 marks)
18 The diagram shows triangle $ABC$. 

$AB = 3.4 \text{ cm} \quad AC = 6.2 \text{ cm} \quad BC = 6.1 \text{ cm}$

$D$ is the point on $BC$ such that

\[
\text{size of angle } DAC = \frac{2}{5} \times \text{size of angle } BCA
\]

Calculate the length $DC$.

Give your answer correct to 3 significant figures.

You must show all your working.

\[
\begin{align*}
\text{.................................................. cm}
\end{align*}
\]

(Total for Question 18 is 5 marks)
19 The graph shows information about part of a cyclist’s journey.

Work out an estimate of the speed, in m/s, of the cyclist at time 6 seconds.

.......................................................

.............................................. m/s

(Total for Question 19 is 3 marks)
20 Here are the first five terms of a sequence.

\[-1 \quad 0 \quad 3 \quad 8 \quad 15\]

Find an expression, in terms of \(n\), for the \(n\)th term of this sequence.

(Total for Question 20 is 2 marks)

21 When a biased coin is thrown 4 times, the probability of getting 4 heads is \(\frac{16}{81}\)

Work out the probability of getting 4 tails when the coin is thrown 4 times.

(Total for Question 21 is 2 marks)
22 Show that \( \frac{7x - 14}{x^2 + 4x - 12} \div \frac{x - 6}{x^3 - 36x} \) simplifies to \( ax \) where \( a \) is an integer.

(Total for Question 22 is 4 marks)
23 The diagram shows a sector $OACB$ of a circle with centre $O$. The point $C$ is the midpoint of the arc $AB$.

The diagram also shows a hollow cone with vertex $O$. The cone is formed by joining $OA$ and $OB$.

The cone has volume $56.8 \text{ cm}^3$ and height $3.6 \text{ cm}$.

Calculate the size of angle $AOB$ of sector $OACB$. Give your answer correct to 3 significant figures. You must show all your working.
24 $OXYZ$ is a parallelogram.

![Diagram of parallelogram OXYZ with points P and R marked on sides OX and OY, respectively.

\[ \overrightarrow{OX} = a \]
\[ \overrightarrow{OY} = b \]

$P$ is the point on $OX$ such that $OP:PX = 1:2$

$R$ is the point on $OY$ such that $OR:RY = 1:3$

Work out, in its simplest form, the ratio $ZP:ZR$

You must show all your working.

(Total for Question 24 is 5 marks)